BME 372 New

Lecture #1

Electrical Basics

Circuit Analysis

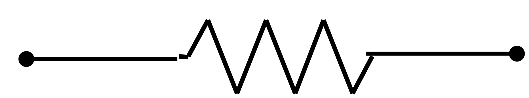
- Circuit Elements
  - Passive Devices
  - Active Devices
- Circuit Analysis Tools
  - Ohms Law
  - Kirchhoff's Law
  - Impedances
  - Mesh and Nodal Analysis
  - Superposition
- Examples

Characterize Circuit Elements

- Passive Devices: dissipates or stores energy
  - Linear
  - Non-linear
- Active Devices: Provider of energy or supports power gain
  - Linear
  - Non-linear

Circuit Elements – Linear Passive Devices

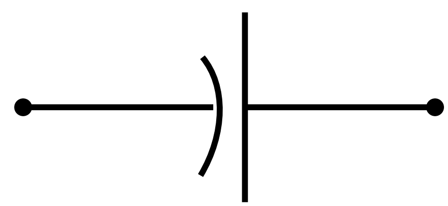
- **Linear**: supports a linear relationship between the voltage across the device and the current through it.
  - **Resistor**: supports a voltage and current which are proportional, device dissipates heat, and is governed by Ohm's Law, units: resistance or ohms Ω



$V_R(t) = I_R(t)R$ where R is the value of the resistance associated with the resistor

Circuit Elements – Linear Passive Devices

– **Capacitor**: supports a current which is proportional to its changing voltage, device stores an electric field between its plates, and is governed by Gauss' Law, units: capacitance or farads, *f*

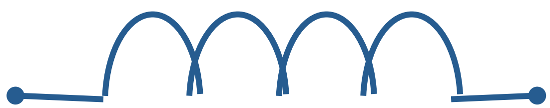


$$I_C(t) = C\frac{dV_C(t)}{dt}$$ where C is the value of the capacitance associated with the capacitor

BME 372 New Schesser

Circuit Elements – Linear Passive Devices

– **Inductor**: supports a voltage which is proportional to its changing current, device stores a magnetic field through its coils and is governed by Faraday's Law, units: inductance or henries, *h*



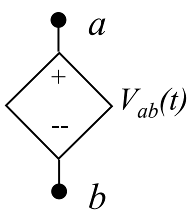
$V_L(t) = L\frac{dI_L(t)}{dt}$ where L is the value of the inductance associated with the inductor

Circuit Elements - Passive Devices Continued

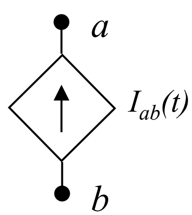
- **Non-linear**: supports a non-linear relationship among the currents and voltages associated with it
  - **Diodes**: supports current flowing through it in only one direction

Circuit Elements - Active Devices

- **Linear**
  - **Sources**
    - **Voltage Source**: a device which supplies a voltage as a function of time at its terminals which is independent of the current flowing through it, units: Volts
      - DC, AC, Pulse Trains, Square Waves, Triangular Waves



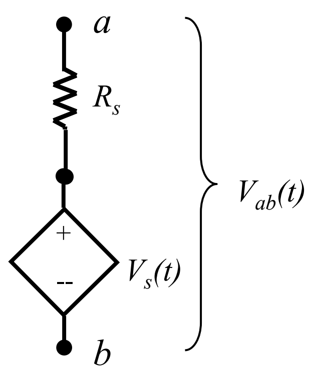
    - **Current Source**: a device which supplies a current as a function of time out of its terminals which is independent of the voltage across it, units: Amperes
      - DC, AC, Pulse Trains, Square Waves, Triangular Waves

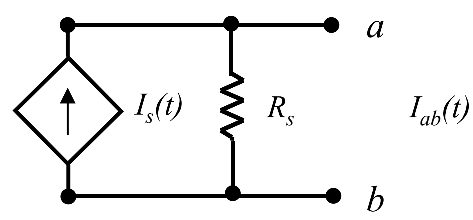


Circuit Elements - Active Devices Continued

– Ideal Sources vs Practical Sources

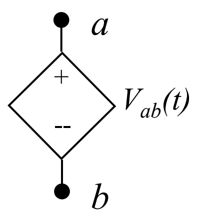
- An ideal source is one which only depends on the type of source (i.e., current or voltage)
- A practical source is one where other circuit elements are associated with it (e.g., resistance, inductance, etc.)
  - – A practical voltage source consists of an ideal voltage source connected in series with passive circuit elements such as a resistor
  - – A practical current source consists of an ideal current source connected in parallel with passive circuit elements such as a resistor



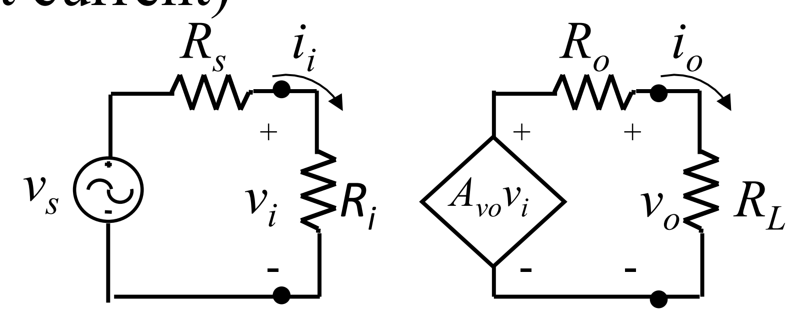


Circuit Elements - Active Devices Continued

– Independent vs Dependent Sources



- An independent source is one where the output voltage or current is not dependent on other voltages or currents in the device
- A dependent source is one where the output voltage or current is a function of another voltage or current in the device (e.g., a BJT transistor may be viewed as having an output current source which is dependent on the input current)



Circuit Elements - Active Devices Continued

- **Non-Linear**
  - **Transistors**: three or more terminal devices where its output voltage and current characteristics are a function on its input voltage and/or current characteristics, several types BJT, FETs, etc.

Circuits

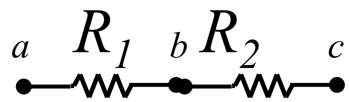
- A circuit is a grouping of passive and active elements
- Elements may be connecting is series, parallel or combinations of both

Circuits Continued

- Series Connection: Same current through the devices
  - The resultant resistance of two or more Resistors connected in series is the sum of the resistance
  - The resultant inductance of two or more Inductors connected in series is the sum of the inductances
  - The resultant capacitance of two or more Capacitors connected in series is the inverse of the sum of the inverse capacitances
  - The resultant voltage of two or more Ideal Voltage Sources connected in series is the sum of the voltages
  - Two of more Ideal Current sources can not be connected in series

Series Circuits

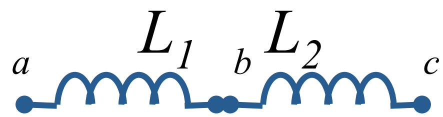
- Resistors



$$R_T = R_1 + R_2$$

$$V_{ac} = V_{ab} + V_{bc} = IR_1 + IR_2$$
$$= I(R_1 + R_2) = IR_T$$

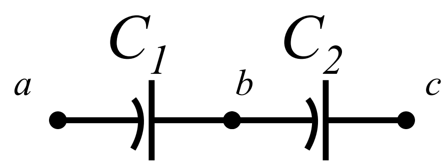
- Inductors



$$L_T = L_1 + L_2$$

$$V_{ac} = V_{ab} + V_{bc} = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt}$$
$$= (L_1 + L_2)\frac{dI}{dt} = L_T \frac{dI}{dt}$$

- Capacitors

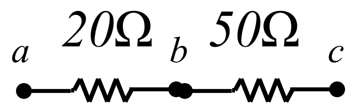


$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

$$V_{ac} = V_{ab} + V_{bc} = \frac{1}{C_1}\int Idt + \frac{1}{C_2}\int Idt$$
$$= (\frac{1}{C_1} + \frac{1}{C_2})\int Idt = \frac{1}{C_T}\int Idt$$

Series Circuits

- Resistors

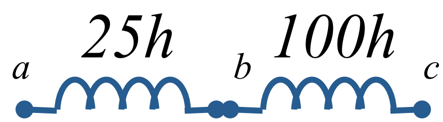


$$R_T = 20 + 50 = 70\Omega$$

$$V_{ac} = V_{ab} + V_{bc} = I20 + I50$$
$$= I(20 + 50) = I70$$

Series Circuits

- Inductors



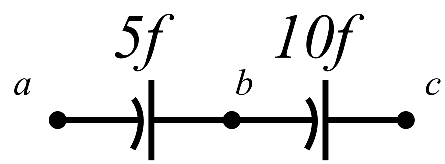
$$L_T = 25 + 100 = 125h$$

$$V_{ac} = V_{ab} + V_{bc} = 25\frac{dI}{dt} + 100\frac{dI}{dt}$$

$$= (25 + 100)\frac{dI}{dt} = 125\frac{dI}{dt}$$

Series Circuits

- Capacitors



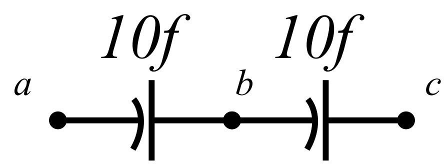
$$C_T = \frac{1}{\frac{1}{5}+\frac{1}{10}} = \frac{5\times 10}{5+10} = \frac{50}{15} = \frac{10}{3} = 3.33f$$

$$V_{ac} = V_{ab} + V_{bc} = \frac{1}{5}\int Idt + \frac{1}{10}\int Idt$$

$$= (\frac{1}{5}+\frac{1}{10})\int Idt = \frac{3}{10}\int Idt$$

Series Circuits

- Capacitors



$$C_T = \frac{1}{\frac{1}{10}+\frac{1}{10}} = \frac{10\times 10}{10+10} = \frac{100}{20} = \frac{10}{2} = 5f$$

$$V_{ac} = V_{ab} + V_{bc} = \frac{1}{10}\int Idt + \frac{1}{10}\int Idt$$

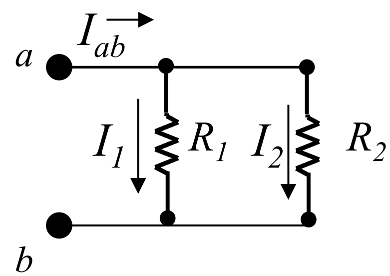
$$= (\frac{1}{10}+\frac{1}{10})\int Idt = \frac{2}{10}\int Idt = \frac{1}{5}\int Idt$$

Circuits Continued

- Parallel Connection: Same Voltage across the devices
  - The resultant resistance of two or more Resistors connected in parallel is the inverse of the sum of the inverse resistances
  - The resultant inductance of two or more Inductors connected in parallel is the inverse of the sum of the inverse inductances
  - The resultant capacitance of two or more Capacitors connected in parallel is the sum of the capacitances
  - The resultant current of two or more Ideal Current Sources connected in parallel is the sum of the currents
  - Two of more Ideal Voltage sources can not be connected in parallel

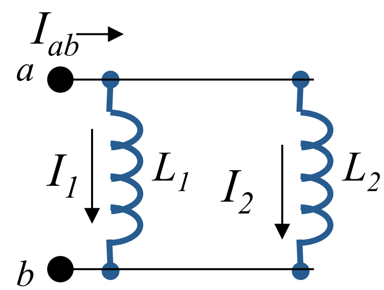
Parallel Circuits

- Resistors $R_T = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \dfrac{R_1 R_2}{R_1 + R_2}$



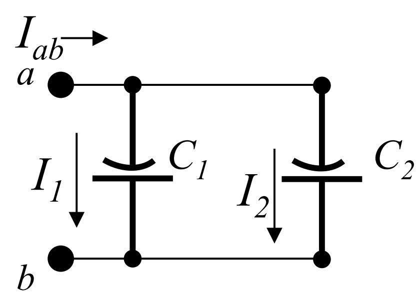
$$I_{ab} = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$$
$$= (\frac{1}{R_1} + \frac{1}{R_2})V = \frac{V}{R_T}$$

- Inductors $L_T = \dfrac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2}} = \dfrac{L_1 L_2}{L_1 + L_2}$



$$I_{ab} = I_1 + I_2 = \frac{1}{L_1}\int V dt + \frac{1}{L_2}\int V dt$$
$$= (\frac{1}{L_1} + \frac{1}{L_2})\int V dt = \frac{1}{L_T}\int V dt$$

- Capacitors $C_T = C_1 + C_2$



$$I_{ab} = I_1 + I_2 = C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt}$$
$$= (C_1 + C_2)\frac{dV}{dt} = C_T \frac{dV}{dt}$$

Combining Circuit Elements Kirchhoff's Laws

- Kirchhoff Voltage Law: The sum of the voltages around a loop must equal zero
- Kirchhoff Current Law: The sum of the currents leaving (entering) a node must equal zero

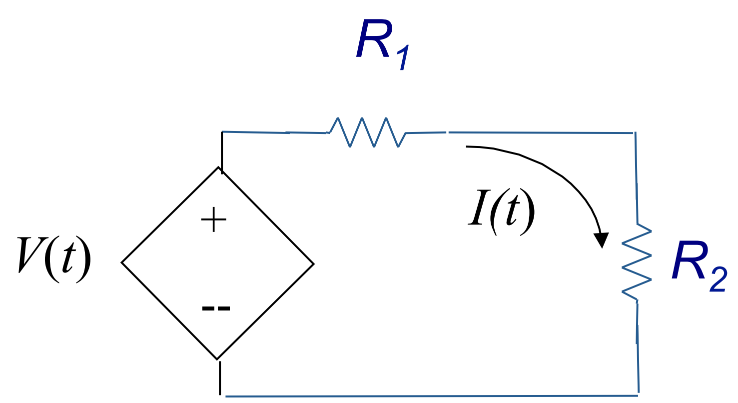
Combining Rs, Ls and Cs

- We can use KVL or KCL to write and solve an equation associated with the circuit.
  - Example: a series Resistive Circuit

Using KVL

$$V(t) = I(t)R_1 + I(t)R_2$$

$$V(t) = I(t)(R_1 + R_2)$$



Combining Rs, Ls, and Cs

- We can use KVL or KCL to write and solve an equation associated with the circuit.
  – Example: a series Resistive Circuit

Using KCL:

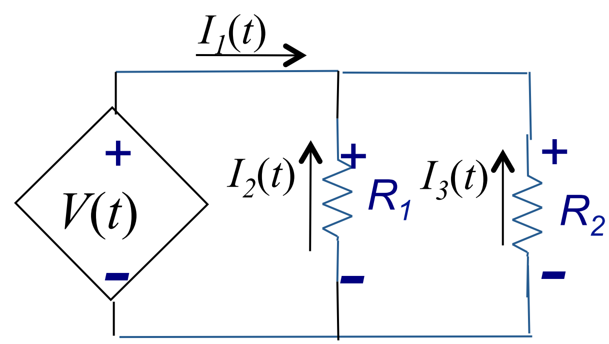
$$I_1 + I_2 + I_3 = 0 \Rightarrow I_1 = -I_2 - I_3$$

But

$$I_2 = -\frac{V(t)}{R_1}; I_3 = -\frac{V(t)}{R_2}$$

Therefore,

$$I_1 = -I_2 - I_3 = \frac{V(t)}{R_1} + \frac{V(t)}{R_2} = [\frac{1}{R_1} + \frac{1}{R_2}]V(t)$$



Impedances

- Our special case, signals of the form: $V(t)$ or $I(t) = Ae^{st}$ where s can be a real or complex number

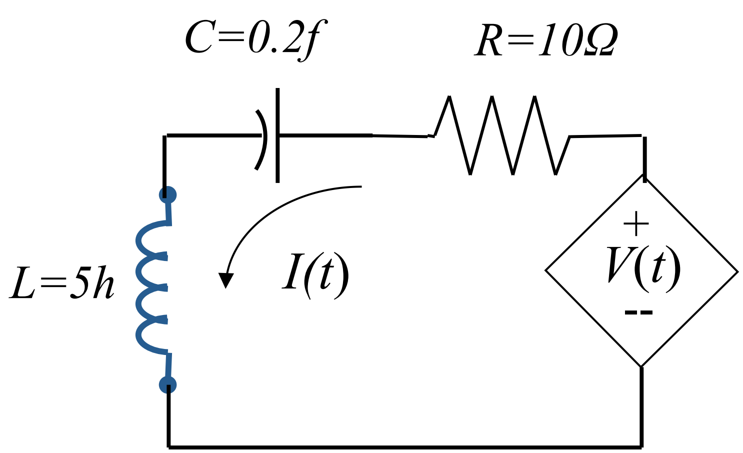
$$V(t) = I(t)R_1 + L_1 \frac{dI(t)}{dt} + \frac{1}{C_1}\int I(t)dt$$

Let's assume:

$$V(t) = 10e^{5t}; R_1 = 10\Omega; L_1 = 5h; C_1 = .2f$$

Let's try:

$$I(t) = Ae^{5t}$$



- This is only one portion of the solution and does not include the transient response.

Impedances

- Our special case, signals of the form: *V(t)* or *I(t)* $=Ae^{st}$ where s can be a real or complex number

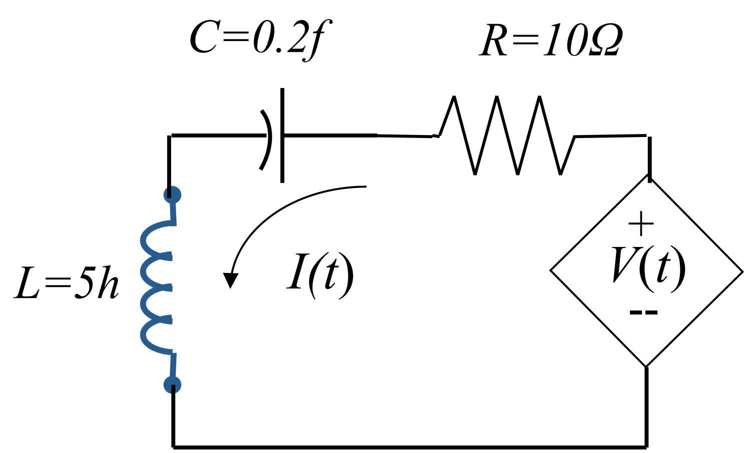
$$10e^{5t} = I(t)10 + 5\frac{dI(t)}{dt} + \frac{1}{.2}\int I(t)dt$$

$$10e^{5t} = Ae^{5t}10 + 5\frac{dAe^{5t}}{dt} + 5\int Ae^{5t}dt$$

$$10e^{5t} = A(e^{5t}10 + 5\times 5e^{5t} + \frac{5e^{5t}}{5})$$

$$= A(e^{5t}36)$$

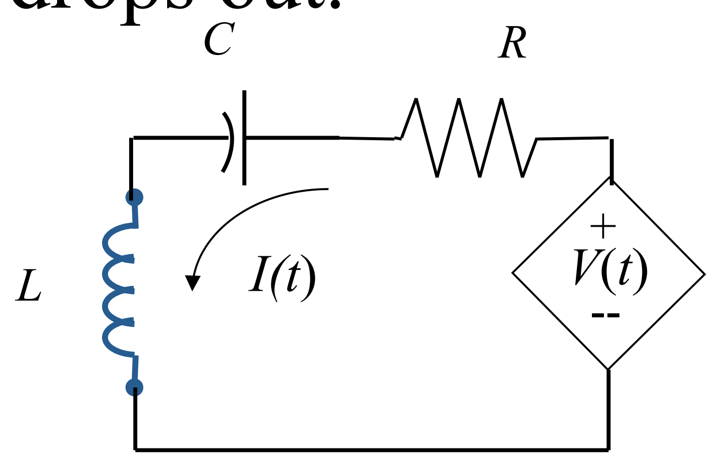
$$A = \frac{10}{36}; I(t) = \frac{10}{36}e^{5t}$$



- This is only one portion of the solution and does not include the transient response.

Impedances

- Since the derivative [and integral] of $Ae^{st} = sAe^{st}$ [$=(1/s)Ae^{st}$], we can define the impedance of a circuit element as $Z(s)=V/I$ where Z is only a function of s since the time dependency drops out.



Impedances

For an inductor, let's assume $I(t) = Ae^{st}$;

then $\quad V_L(t) = L\dfrac{dI(t)}{dt} = LsAe^{st}$;

$$Z(s) = \frac{V_L}{I} = \frac{sLAe^{st}}{Ae^{st}} = sL$$

For a capacitor, let's assume $V(t) = Ae^{st}$;

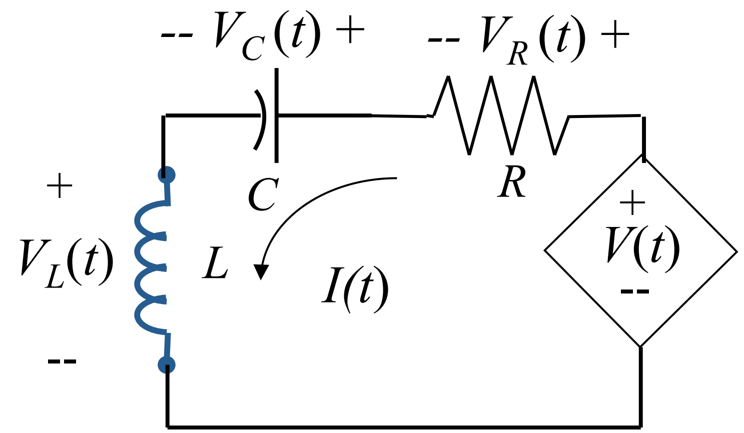
then $\quad I(t) = C\dfrac{dV_C(t)}{dt} = CsAe^{st}$;

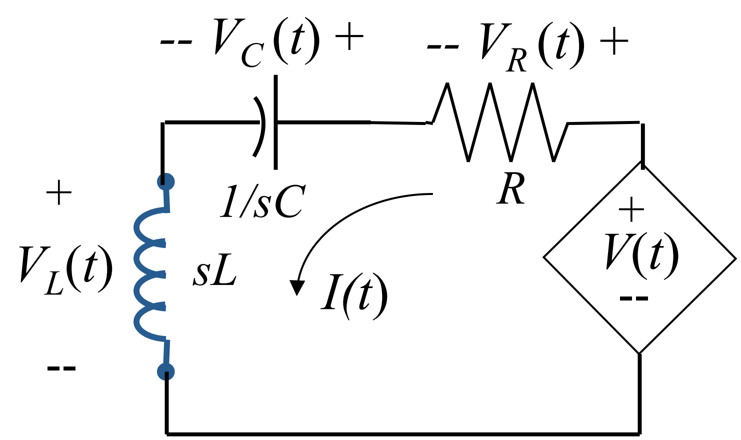
$$Z(s) = \frac{V_C}{I} = \frac{Ae^{st}}{sCAe^{st}} = \frac{1}{sC}$$

For a resistor, let's assume $I(t) = Ae^{st}$;

then $\quad V_R(t) = RI(t) = RAe^{st}$;

$$Z(s) = \frac{V_R}{I} = \frac{RAe^{st}}{Ae^{st}} = R$$





Impedances

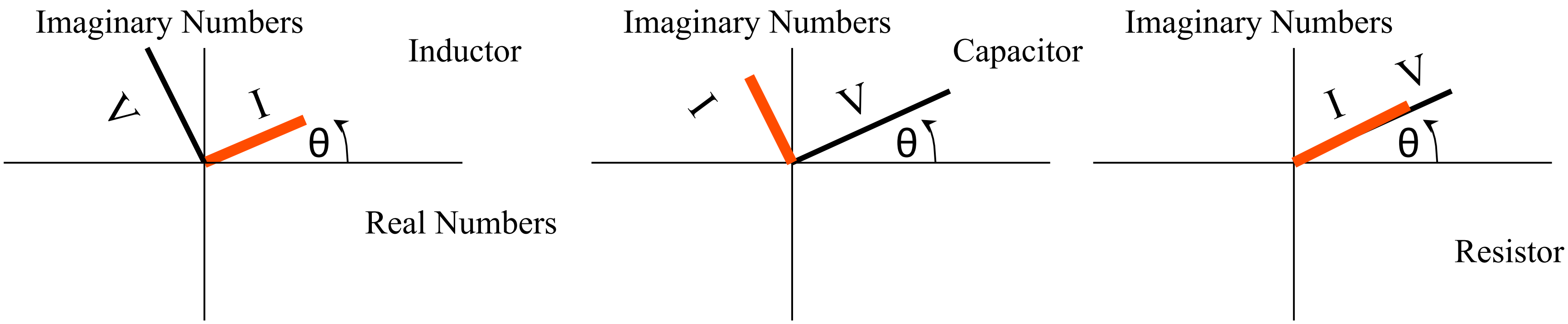
- What about signals of the type: $\cos(\omega t+\theta)$;

- Recall Euler's formula $e^{j\theta} = \cos\theta + j\sin\theta$ where j is the imaginary number = $\sqrt{-1}$

- A special case of our special case is for sinusoidal inputs, where $s=j\omega$

Sinusoidal Steady State Continued

- For an inductor, $Z_L = j\omega L \Rightarrow \omega L \angle \frac{\pi}{2}$
- For a capacitor, $Z_C = \frac{1}{j\omega C} \Rightarrow \frac{1}{\omega C} \angle -\frac{\pi}{2}$.
- For a resistor, $Z_R = R \Rightarrow R\angle 0$



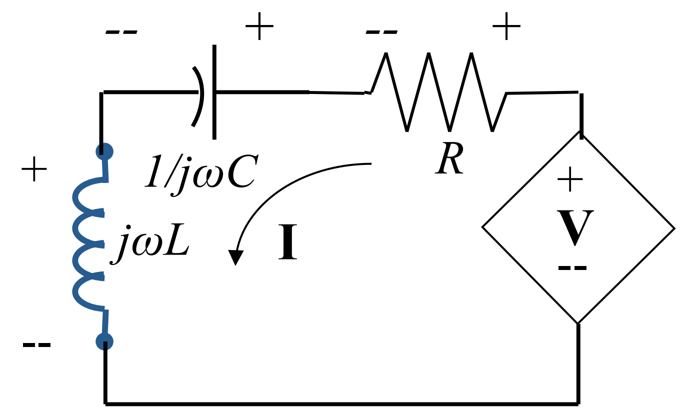
Sinusoidal Steady State Continued

For $V(t)=A\cos\omega t$, using phasor notation for $V(t)$ ➔ $\mathbf{V} = A\angle 0$ and $I(t)$ ➔ $\mathbf{I}$, our equation can be re-written:

$$V(t) = I(t)R_1 + L_1\frac{dI(t)}{dt} + \frac{1}{C_1}\int I(t)dt$$

Converting to Phasor representation

$$\mathbf{V} = A\angle 0 = \mathbf{I}R_1 + j\omega L_1\mathbf{I} + \frac{1}{j\omega C_1}\mathbf{I}$$



Sinusoidal Steady State Continued

$$\mathbf{I} = \frac{A\angle 0}{R_1 + j\omega L_1 + \dfrac{1}{j\omega C_1}} = \frac{A\angle 0}{R_1 + j(\omega L_1 - \dfrac{1}{\omega C_1})} = \frac{A}{\sqrt{R_1^2 + (\omega L_1 - \dfrac{1}{\omega C_1})^2}} \angle -\tan^{-1}[\frac{(\omega L_1 - \dfrac{1}{\omega C_1})}{R_1}]$$

Converting back to the time representation,

$$I(t) = \frac{A}{\sqrt{R_1^2 + (\omega L_1 - \dfrac{1}{\omega C_1})^2}} \cos(\omega t - \tan^{-1}[\frac{(\omega L_1 - \dfrac{1}{\omega C_1})}{R_1}])$$

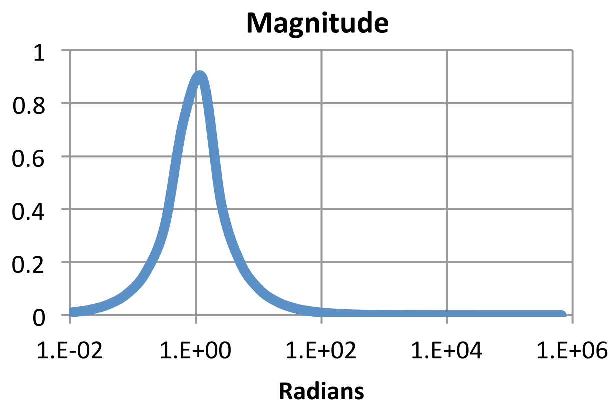
Bode Plots

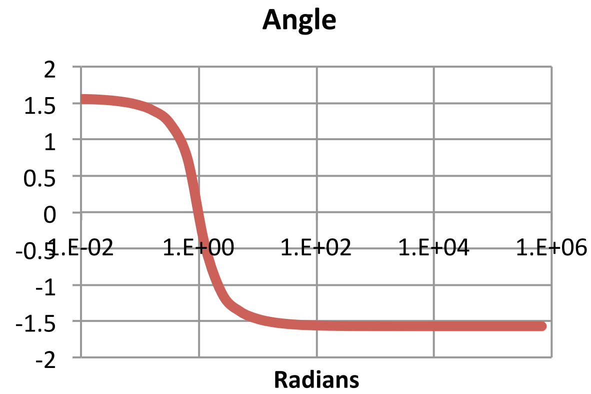
- Plotting the magnitude and phase versus frequency.

$$\mathbf{I} = \frac{A}{\sqrt{R_1^2 + (\omega L_1 - \frac{1}{\omega C_1})^2}} \angle -\tan^{-1}\left[\frac{(\omega L_1 - \frac{1}{\omega C_1})}{R_1}\right]$$

$$\text{Magnitude of I} \Rightarrow |\mathbf{I}| = \frac{A}{\sqrt{R_1^2 + (\omega L_1 - \frac{1}{\omega C_1})^2}}$$

$$\text{Angle of I} \Rightarrow \angle\mathbf{I} = \angle -\tan^{-1}\left[\frac{(\omega L_1 - \frac{1}{\omega C_1})}{R_1}\right]$$





Bode Plots

- Easy way to plot the magnitude.

$$\mathbf{I} = \frac{A}{R_1 + j\omega L_1 + \frac{1}{j\omega C_1}}$$

First evaluate at the initial condition ω=0

$$\mathbf{I}|_{\omega=0} = \frac{A}{R_1 + j\omega L_1 + \frac{1}{j\omega C_1}}|_{\omega=0} \rightarrow \frac{A}{R_1 + j0L_1 + \frac{1}{j0C_1}} \underset{\text{Determine the dominate terms}}{\rightarrow} \frac{A}{\frac{1}{j0C_1}} \rightarrow j\frac{A}{\infty} \rightarrow 0\angle\frac{\pi}{2}$$

Next evaluate at the final condition $\omega \rightarrow \infty$

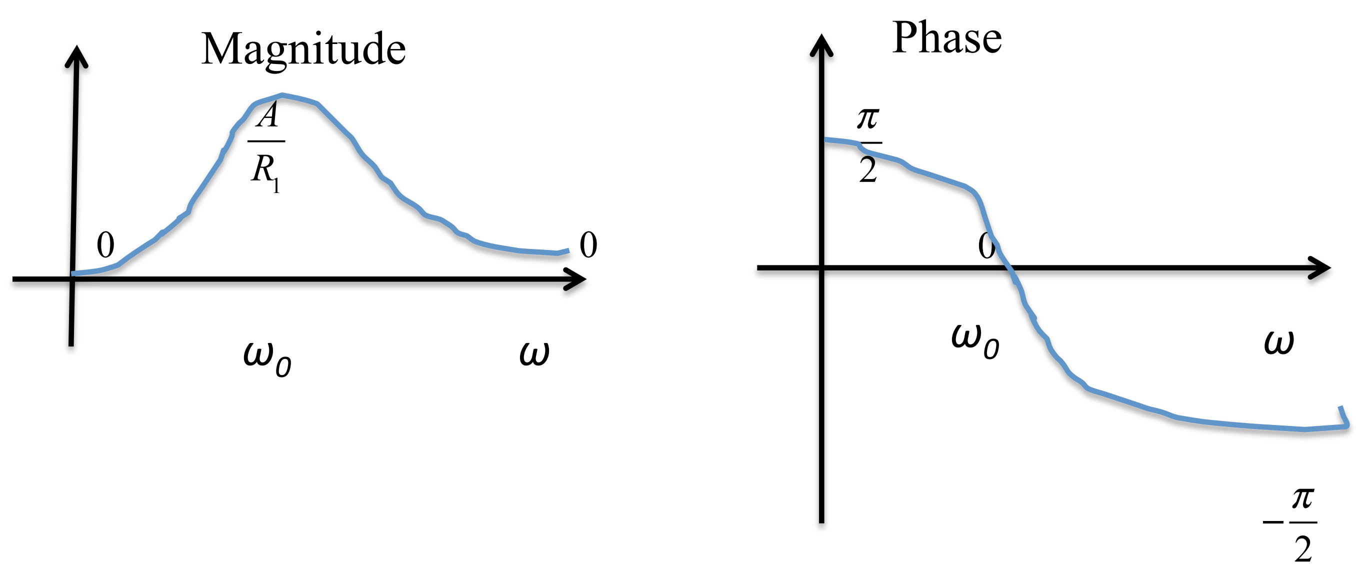
$$\mathbf{I}|_{\omega\rightarrow\infty} = \frac{A}{R_1 + j\omega L_1 + \frac{1}{j\omega C_1}}|_{\omega\rightarrow\infty} \rightarrow \frac{A}{R_1 + j\infty L_1 + \frac{1}{j\infty C_1}} \underset{\text{Determine the dominate terms}}{\rightarrow} \frac{A}{j\infty L_1} \rightarrow -j\frac{A}{\infty} \rightarrow 0\angle-\frac{\pi}{2}$$

Find an interesting point; for this example choose $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$ since this is when the imaginary part of the deminator is zero.

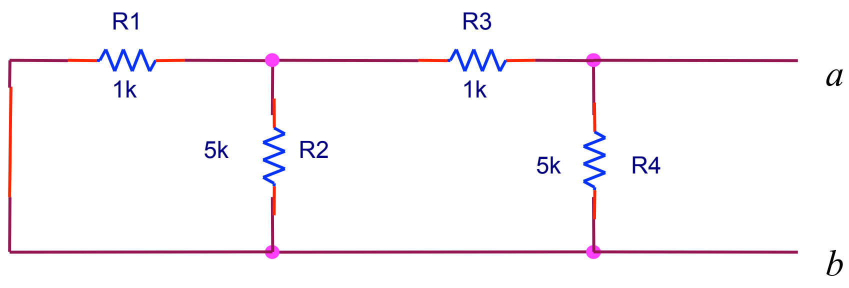
$$\mathbf{I}|_{\omega_0=\frac{1}{\sqrt{LC}}} = \frac{A}{R_1 + j\omega L_1 + \frac{1}{j\omega C_1}}|_{\omega_0=\frac{1}{\sqrt{L_1C_1}}} = \frac{A}{R_1 + j(\frac{1}{\sqrt{L_1C_1}}L_1 - \frac{1}{\frac{1}{\sqrt{L_1C_1}}C_1})} = \frac{A}{R_1 + j(\sqrt{\frac{L_1}{C_1}} - \frac{1}{\sqrt{\frac{C_1}{L_1}}})} = \frac{A}{R_1} = \frac{A}{R_1}\angle 0$$

Bode Plots

- With the three point, plots can be made



Homework

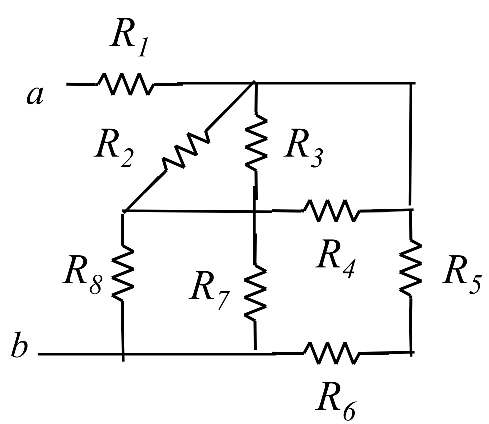


Find the total resistance

Homework

Find the total resistance R_{ab} where

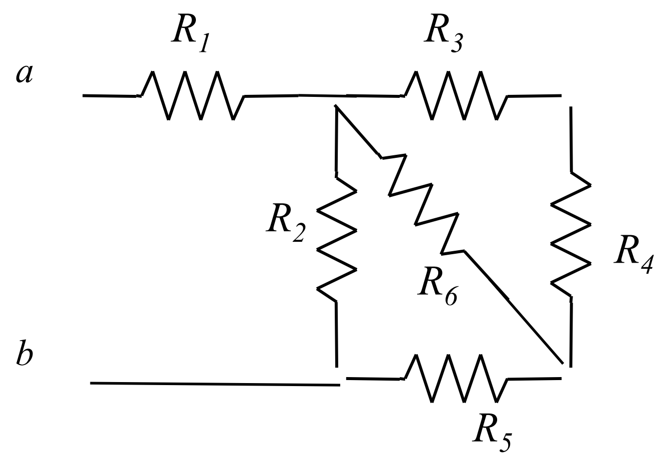
$R_1 = 3\Omega$, $R_2 = 6\Omega$, $R_3 = 12\Omega$, $R_4 = 4\Omega$, $R_5 = 2\Omega$, $R_6 = 2\Omega$, $R_7 = 4\Omega$, $R_8 = 4\Omega$



Homework

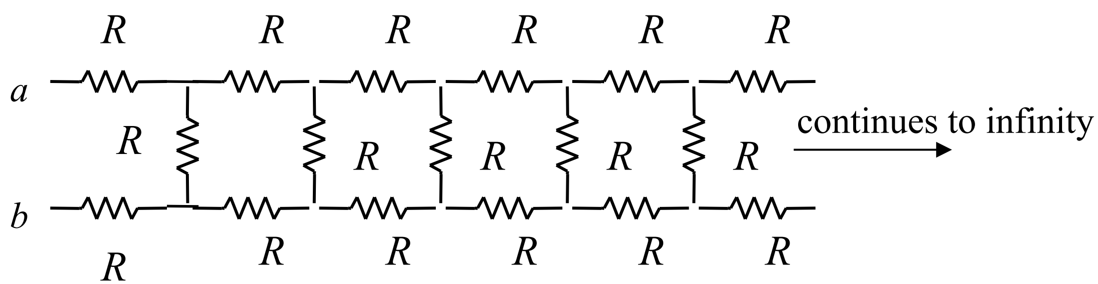
Find the total resistance R_{ab} where

$R_1 = 2\Omega$, $R_2 = 4\Omega$, $R_3 = 2\Omega$, $R_4 = 2\Omega$, $R_5 = 2\Omega$, $R_6 = 4\Omega$,

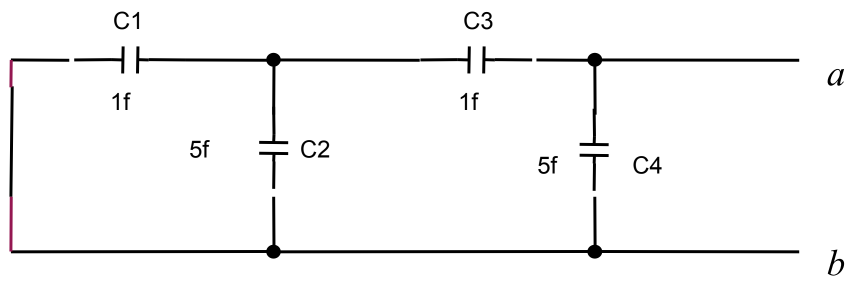


Homework

Find the total resistance R_{ab} for this infinite resistive network



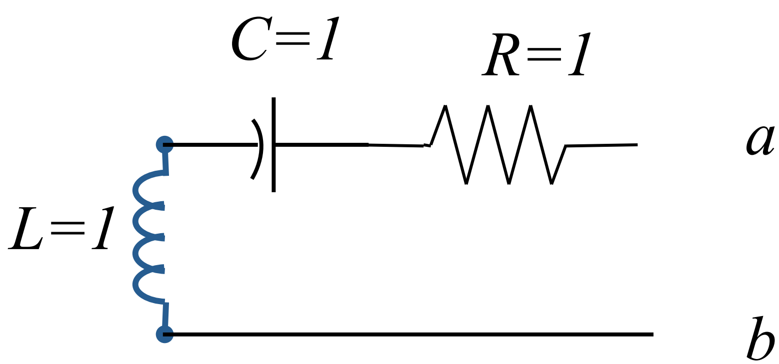
Homework



Find the total capacitance

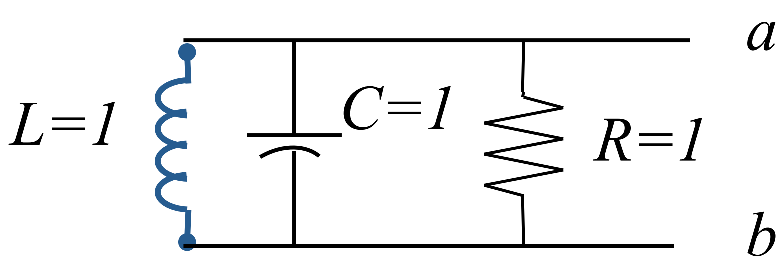
Homework

Find and plot the impedance $Z_{ab}(j\omega)$ as a function of frequency. Use Matlab to calculate the Bode plot.

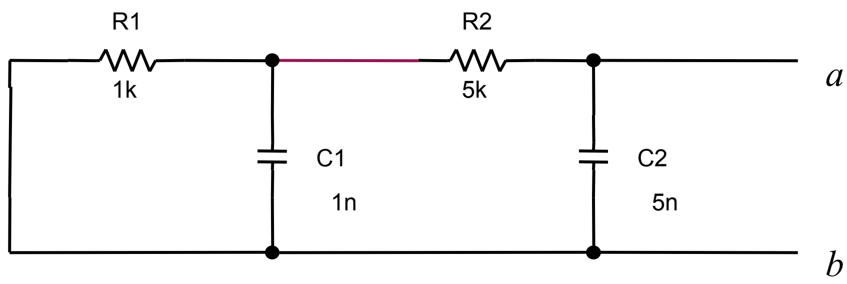


Homework

Find and plot the impedance $Z_{ab}(j\omega)$ as a function of frequency. Use Matlab to calculate the Bode plot.



Homework



Find and plot the impedance $Z_{ab}(j\omega)$ as function of ω.
Use Matlab to calculate the Bode plot.